AlKarkh University of Science
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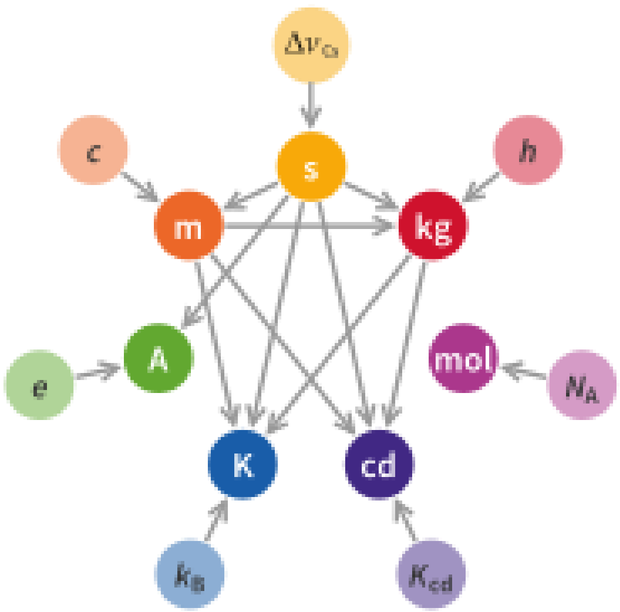

Fluid Mechanics

Stage-2

LECTURE ONE DIMENSIONS AND UNITS

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2022

LECTURE ONE

Dimensions and Units

<u>Definitions of Fluid Mechanics</u>

Is the study of fluids either in motion (fluid *dynamics*) or at rest (fluid *statics*). Both gases and liquids are classified as fluids, and the number of fluid engineering applications is enormous: breathing, blood flow, swimming, pumps, fans, turbines, airplanes, ships, rivers, windmills, pipes, missiles, icebergs, engines, filters, jets, and sprinklers, to name a few. Therefore, can be define the **fluid** is a substance that deforms continuously when subjected to shear stress. It is either gas or liquid.

<u>Dimensions and Units</u>

<u>Dimensions:</u>

A *dimension* is the measure by which a physical variable is expressed quantitatively. **Example,** length is a dimension associated with such variables as distance, displacement, width, deflection, and height.

Dimension is a powerful concept about which a splendid tool called *dimensional analysis*.

<u>Units:</u>

A *unit* is a particular way of attaching a number to the quantitative dimension. **Example,** centimeters and inches are both numerical units for expressing length. units are the numerical quantity that the customer wants as the final answer.

Dimensions types

1. Primary Dimensions: The SI (International System of Units) consists of **six** primary units, which are **length**, **mass**, **time**, **temperature**, **current** and **luminosity**. In fluid mechanics there are only **four** primary dimensions (as shown in Table 1.1) from which all other dimensions can be derived: **mass**, **length**, **time**, and **temperature**. British gravitational (BG) units will also be presented in this lecture.

Table 1.1: primary dimensions in SI and BG systems

Primary dimension	SI unit	BG unit	Conversion factor
Mass $\{M\}$	Kilogram (kg)	Slug	1 slug = 14.5939 kg
Length $\{L\}$	Meter (m)	Foot (ft)	1 ft = 0.3048 m
Time $\{T\}$	Second (s)	Second (s)	1 s = 1 s
Temperature $\{\Theta\}$	Kelvin (K)	Rankine (°R)	1 K = 1.8°R

Note that the kelvin unit uses no degree symbol. The braces around a symbol like **{M}** mean "the **dimension**" of mass. All other variables in fluid mechanics can be expressed in terms of **{M}, {L}, {T}, and {Θ}**. For example, acceleration has the dimensions $\{LT^{-2}\}$. The most crucial of these secondary dimensions is force, which is directly related to mass, length, and time by Newton's second law. Force equals the time rate of change of momentum or, for constant mass,

$$F = ma \qquad (1)$$

From this we see that, dimensionally, $\{F\} = \{MLT^{-2}\}$.

SI System

The use of a constant of proportionality in Newton's law, Eq. (1), is avoided by defining the force unit exactly in terms of the other basic units. In the SI system, the basic units are newtons {F}, kilograms {M}, meters {L}, and seconds {T}. We define

$$1 \text{ newton of force} = \mathbf{1\ N = 1\ kg\ 1\ m/s^2}$$

The newton is a relatively small force, about the weight of an apple (0.225 lbf). In addition, the basic unit of temperature {Θ} in the SI system is the degree Kelvin, K. Use of these SI units (N, kg, m, s, K) will require no conversion factors in our equations.

BG System

In the BG system also, a constant of proportionality in Eq. (1) is avoided by defining the force unit exactly in terms of the other basic units. In the BG system, the basic units are pound-force {F}, slugs {M}, feet {L}, and seconds {T}. We define

1 pound of force $\quad\quad$ $\mathbf{1\ lbf = 1\ slug.\ 1\ ft/s^2}$

One lbf ≈ 4.4482 N and approximates the weight of four apples. We will use the abbreviation lbf for pound-force and lbm for pound-mass. The slug is a rather hefty mass, equal to 32.174 lbm. The basic unit of temperature {} in the BG system is the degree Rankine, R. Recall that a temperature difference 1 K = 1.8 ^{o}R. Use of these BG units (lbf, slug, ft, s, ^{o}R) will require no conversion factors in our equations.

Other Unit systems

There are other unit systems still in use. At least one needs no proportionality constant: the CGS system (dyne, gram, cm, s, K). However, CGS units are too small for most applications (1 dyne = 10^{-5} N) and will not be used here.

In the USA, some still use the English Engineering system, (lbf, lbm, ft, s, R), where the basic mass unit is the pound of mass. Newton's law (1) must be rewritten:

$$F = \frac{ma}{g_c} \qquad \text{where} \qquad g_c = 32.174\ \frac{ft.lb_m}{lb_f.\ S^2}$$

In engineering and science, *all* equations must be *dimensionally homogeneous*, that is, each additive term in an equation must have the same dimensions. For example, take **Bernoulli's incompressible equation**, to be studied and used throughout this text:

$$P + \frac{1}{2}\rho V^2 + \rho g z = constant$$

Each and every term in this equation *must* have dimensions of pressure $\{ML1T^{-2}\}$. We will examine the dimensional homogeneity of this equation in detail in **Ex.1.3**.

2. Derived (Secondary) dimensions

For conveniences secondary units are used in general practice which are made from combinations of these primary units as shown in Table 1.2.

Table 1.2: secondary dimensions in fluid mechanics

Secondary dimension	SI unit	BG unit	Conversion factor
Area $\{L^2\}$	m^2	ft^2	$1\ m^2 = 10.764\ ft^2$
Volume $\{L^3\}$	m^3	ft^3	$1\ m^3 = 35.315\ ft^3$
Velocity $\{LT^{-1}\}$	m/s	ft/s	1 ft/s = 0.3048 m/s
Acceleration $\{LT^{-2}\}$	m/s^2	ft/s^2	$1\ ft/s^2 = 0.3048\ m/s^2$
Pressure or stress $\{ML^{-1}T^{-2}\}$	$Pa = N/m^2$	lbf/ft^2	$1\ lbf/ft^2 = 47.88\ Pa$
Angular velocity $\{T^{-1}\}$	s^{-1}	s^{-1}	$1\ s^{-1} = 1\ s^{-1}$
Energy, heat, work $\{ML^2T^{-2}\}$	$J = N \cdot m$	$ft \cdot lbf$	$1\ ft \cdot lbf = 1.3558\ J$
Power $\{ML^2T^{-3}\}$	W = J/s	$ft \cdot lbf/s$	$1\ ft \cdot lbf/s = 1.3558\ W$
Density $\{ML^{-3}\}$	kg/m^3	$slugs/ft^3$	$1\ slug/ft^3 = 515.4\ kg/m^3$
Viscosity $\{ML^{-1}T^{-1}\}$	$kg/(m \cdot s)$	$slugs/(ft \cdot s)$	$1\ slug/(ft \cdot s) = 47.88\ kg/(m \cdot s)$
Specific heat $\{L^2T^{-2}\Theta^{-1}\}$	$m^2/(s^2 \cdot K)$	$ft^2/(s^2 \cdot °R)$	$1\ m^2/(s^2 \cdot K) = 5.980\ ft^2/(s^2 \cdot °R)$

EXAMPLE 1.1

A body weighs 1000 lbf when exposed to a standard earth gravity $g = 32.174$ ft/s^2. (*a*) What is its mass in kg? (*b*) What will the weight of this body be in N if it is exposed to the moon's standard acceleration $g_{moon} = 1.62$ m/s^2? (*c*) How fast will the body accelerate if a net force of 400 lbf is applied to it on the moon or on the earth?

Solution

We need to find the (*a*) mass; (*b*) weight on the moon; and (*c*) acceleration of this body. This is a fairly simple example of conversion factors for differing unit systems. No property data is needed. The example is too low-level for a sketch.

Part (a) Newton's law (1.2) holds with known weight and gravitational acceleration. Solve for *m:*

$$F = W = 1000 \text{ lbf} = mg = (m)(32.174 \text{ ft/s}^2), \quad \textit{or} \quad m = \frac{1000 \text{ lbf}}{32.174 \text{ ft/s}^2} = 31.08 \text{ slugs}$$

Convert this to kilograms:

$$m = 31.08 \text{ slugs} = (31.08 \text{ slugs})(14.5939 \text{ kg/slug}) = 454 \text{ kg} \qquad \textit{Ans. (a)}$$

Part (b) The mass of the body remains 454 kg regardless of its location. Equation (1.2) applies with a new gravitational acceleration and hence a new weight:

$$F = W_{moon} = mg_{moon} = (454 \text{ kg})(1.62 \text{ m/s}^2) = 735 \text{ N} \qquad \textit{Ans. (b)}$$

Part (c) This part does not involve weight or gravity or location. It is simply an application of Newton's law with a known mass and known force:

$$F = 400 \text{ lbf} = ma = (31.08 \text{ slugs})\ a$$

Solve for

$$a = \frac{400 \text{ lbf}}{31.08 \text{ slugs}} = 12.87\frac{\text{ft}}{\text{s}^2}\left(0.3048\frac{\text{m}}{\text{ft}}\right) = 3.92\frac{\text{m}}{\text{s}^2} \qquad \textit{Ans. (c)}$$

Comment (c): This acceleration would be the same on the earth or moon or anywhere.

Many data in the literature are reported in inconvenient or arcane units suitable only to some industry or specialty or country. The engineer should convert these data to the SI or BG system before using them. This requires the systematic application of conversion factors, as in the following example.

EXAMPLE 1.2

Industries involved in viscosity measurement [27, 36] continue to use the CGS system of units, since centimeters and grams yield convenient numbers for many fluids. The absolute viscosity (μ) unit is the *poise,* named after J. L. M. Poiseuille, a French physician who in 1840 performed pioneering experiments on water flow in pipes; 1 poise = 1 g/(cm-s). The kinematic viscosity (ν) unit is the *stokes,* named after G. G. Stokes, a British physicist who in 1845 helped develop the basic partial differential equations of fluid momentum; 1 stokes = 1 cm^2/s. Water at 20°C has $\mu \approx 0.01$ poise and also $\nu \approx 0.01$ stokes. Express these results in (*a*) SI and (*b*) BG units.

Solution

Part (a)

- *Approach:* Systematically change grams to kg or slugs and change centimeters to meters or feet.
- *Property values:* Given $\mu = 0.01$ g/(cm-s) and $\nu = 0.01$ cm^2/s.
- *Solution steps:* (a) For conversion to SI units,

$$\mu = 0.01 \frac{g}{cm \cdot s} = 0.01 \frac{g(1\ kg/1000g)}{cm(0.01\ m/cm)s} = 0.001 \frac{kg}{m \cdot s}$$

$$\nu = 0.01 \frac{cm^2}{s} = 0.01 \frac{cm^2(0.01\ m/cm)^2}{s} = 0.000001 \frac{m^2}{s} \qquad \textit{Ans. (a)}$$

Part (b)

- For conversion to BG units

$$\mu = 0.01 \frac{g}{cm \cdot s} = 0.01 \frac{g(1\ kg/1000\ g)(1\ slug/14.5939\ kg)}{(0.01\ m/cm)(1\ ft/0.3048\ m)s} = 0.0000209 \frac{slug}{ft \cdot s}$$

$$\nu = 0.01 \frac{cm^2}{s} = 0.01 \frac{cm^2(0.01\ m/cm)^2(1\ ft/0.3048\ m)^2}{s} = 0.0000108 \frac{ft^2}{s} \qquad \textit{Ans. (b)}$$

- *Comments:* This was a laborious conversion that could have been shortened by using the direct viscosity conversion factors in App. C. For example, $\mu_{BG} = \mu_{SI}/47.88$.

EXAMPLE 1.3

A useful theoretical equation for computing the relation between pressure, velocity, and altitude in a steady flow of a nearly inviscid, nearly incompressible fluid with negligible heat transfer and shaft work[5] is the *Bernoulli relation,* named after Daniel Bernoulli, who published a hydrodynamics textbook in 1738:

$$p_0 = p + \tfrac{1}{2}\rho V^2 + \rho g Z \qquad (1)$$

where p_0 = stagnation pressure
p = pressure in moving fluid
V = velocity
ρ = density
Z = altitude
g = gravitational acceleration

(*a*) Show that Eq. (1) satisfies the principle of dimensional homogeneity, which states that all additive terms in a physical equation must have the same dimensions. (*b*) Show that consistent units result without additional conversion factors in SI units. (*c*) Repeat (*b*) for BG units.

Solution

Part (a)

We can express Eq. (1) dimensionally, using braces, by entering the dimensions of each term from Table 1.2:

$$\{ML^{-1}T^{-2}\} = \{ML^{-1}T^{-2}\} + \{ML^{-3}\}\{L^2T^{-2}\} + \{ML^{-3}\}\{LT^{-2}\}\{L\}$$
$$= \{ML^{-1}T^{-2}\} \text{ for all terms} \qquad \textit{Ans. (a)}$$

Part (b)

Enter the SI units for each quantity from Table 1.2:

$$\{\text{N/m}^2\} = \{\text{N/m}^2\} + \{\text{kg/m}^3\}\{\text{m}^2/\text{s}^2\} + \{\text{kg/m}^3\}\{\text{m/s}^2\}\{\text{m}\}$$
$$= \{\text{N/m}^2\} + \{\text{kg/(m}\cdot\text{s}^2)\}$$

The right-hand side looks bad until we remember from Eq. (1.3) that 1 kg = 1 N · s²/m.

$$\{\text{kg/(m}\cdot\text{s}^2)\} = \frac{\{\text{N}\cdot\text{s}^2/\text{m}\}}{\{\text{m}\cdot\text{s}^2\}} = \{\text{N/m}^2\} \qquad \textit{Ans. (b)}$$

Thus all terms in Bernoulli's equation will have units of pascals, or newtons per square meter, when SI units are used. No conversion factors are needed, which is true of all theoretical equations in fluid mechanics.

Part (c) Introducing BG units for each term, we have

$$\{\text{lbf/ft}^2\} = \{\text{lbf/ft}^2\} + \{\text{slugs/ft}^3\}\{\text{ft}^2/\text{s}^2\} + \{\text{slugs/ft}^3\}\{\text{ft/s}^2\}\{\text{ft}\}$$
$$= \{\text{lbf/ft}^2\} + \{\text{slugs/(ft}\cdot\text{s}^2)\}$$

But, from Eq. (1.3), 1 slug = 1 lbf $\cdot$ s^2/ft, so that

$$\{\text{slugs/(ft}\cdot\text{s}^2)\} = \frac{\{\text{lbf}\cdot\text{s}^2/\text{ft}\}}{\{\text{ft}\cdot\text{s}^2\}} = \{\text{lbf/ft}^2\} \qquad \textit{Ans. (c)}$$

All terms have the unit of pounds-force per square foot. No conversion factors are needed in the BG system either.

There is still a tendency in English-speaking countries to use pound-force per square inch as a pressure unit because the numbers are more manageable. For example, standard atmospheric pressure is 14.7 lbf/in^2 = 2116 lbf/ft^2 = 101,300 Pa. The pascal is a small unit because the newton is less than $\frac{1}{4}$ lbf and a square meter is a very large area.

Convenient Prefixes in power 10:

Engineering results often are too small or too large for the common units, with too many zeros one way or the other. For example, to write p = 114,000,000 Pa is long and awkward. Using the prefix "M" to mean 10^6, we convert this to a concise p =114 MPa (megapascals). Similarly, t = 0.000000003 s is a proofreader's nightmare compared to the equivalent t = 3 ns (nanoseconds). Such prefixes are common and convenient, in both the SI and BG systems. A complete list is given in Table 1.3.

Table 1.3: convenient prefixes for engineering units

Multiplicative factor	Prefi	Symbol			
10^{12}	tera	T	10^{-1}	deci	d
10^{9}	giga	G	10^{-2}	centi	c
10^{6}	mega	M	10^{-3}	milli	m
10^{3}	kilo	k	10^{-6}	micro	μ
10^{2}	hecto	h	10^{-9}	nano	n
10	deka	da	10^{-12}	pico	p
			10^{-15}	femto	f
			10^{-18}	atto	a

EXAMPLE 1.4

In 1890 Robert Manning, an Irish engineer, proposed the following empirical formula for the average velocity V in uniform flow due to gravity down an open channel (BG units):

$$V = \frac{1.49}{n} R^{2/3} S^{1/2} \qquad (1)$$

where R = hydraulic radius of channel (Chaps. 6 and 10)
S = channel slope (tangent of angle that bottom makes with horizontal)
n = Manning's roughness factor (Chap. 10)

and n is a constant for a given surface condition for the walls and bottom of the channel. (*a*) Is Manning's formula dimensionally consistent? (*b*) Equation (1) is commonly taken to be valid in BG units with n taken as dimensionless. Rewrite it in SI form.

Solution

- *Assumption:* The channel slope S is the tangent of an angle and is thus a dimensionless ratio with the dimensional notation $\{1\}$—that is, not containing M, L, or T.
- *Approach (a):* Rewrite the dimensions of each term in Manning's equation, using brackets $\{\}$:

$$\{V\} = \left\{\frac{1.49}{n}\right\}\{R^{2/3}\}\{S^{1/2}\} \quad \text{or} \quad \left\{\frac{L}{T}\right\} = \left\{\frac{1.49}{n}\right\}\{L^{2/3}\}\{1\}$$

This formula is incompatible unless $\{1.49/n\} = \{L^{1/3}/T\}$. If n is dimensionless (and it is never listed with units in textbooks), the number 1.49 must carry the dimensions of $\{L^{1/3}/T\}$. *Ans.* (*a*)

- *Comment (a):* Formulas whose numerical coefficients have units can be disastrous for engineers working in a different system or another fluid. Manning's formula, though popular, is inconsistent both dimensionally and physically and is valid only for water flow with certain wall roughnesses. The effects of water viscosity and density are hidden in the numerical value 1.49.
- *Approach (b):* Part (*a*) showed that 1.49 has dimensions. If the formula is valid in BG units, then it must equal 1.49 ft$^{1/3}$/s. By using the SI conversion for length, we obtain

$$(1.49\ \text{ft}^{1/3}/\text{s})(0.3048\ \text{m/ft})^{1/3} = 1.00\ \text{m}^{1/3}/\text{s}$$

Therefore Manning's inconsistent formula changes form when converted to the SI system:

$$\text{SI units:} \quad V = \frac{1.0}{n} R^{2/3} S^{1/2} \qquad \textit{Ans. (b)}$$

with R in meters and V in meters per second.

- *Comment (b):* Actually, we misled you: This is the way Manning, a metric user, first proposed the formula. It was later converted to BG units. Such dimensionally inconsistent formulas are dangerous and should either be reanalyzed or treated as having very limited application.

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LECTURE TWO
FLUID PROPERTIES

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FLUID PROPERTIES

Property is a characteristic of a system. It may either be intensive (mass independent) or extensive (that depends on size of system). The most common properties of the fluid are:

Pressure (P): It is the normal force exerted by a fluid per unit area (F/A). In SI system the unit and dimension of pressure can be written as, N/m^2 and $ML\text{-}1T^{-2}$, respectively.

Density (ρ):

mass density (ρ=mass / volume), specific volume ($\nu=1/\rho$)

specific weight ($\gamma=\rho g$), represents the force exerted by gravity on a unit volume of fluid

relative density (specific gravity) $SGliquid=\rho_{liquid}/\rho_{water}$,

$$SGgas=\rho_{gas}/\rho_{air}$$

The units and dimensions are given as,

For mass density; Dimension: ML^{-3} Unit: kg/m^3

For specific weight; Dimension: $ML^{-2}T^{-2}$ Unit: N/m^3

For specific gravity; Dimensionless

Notes: ρ_{water} = 1000 kg/m^3 , ρ_{air} = 1.2 kg/m^3

Example 2.1: The specific weight of water at ordinary pressure and temperature is 9.81 kN/m^3 . The specific gravity of mercury is 13.56. Compute the density of water and the specific weight and density of mercury.

Solution:

$$\rho water = \frac{\gamma water}{g} = \frac{9.81\ kN/m3}{9.81\,m/s2} = 1.00\frac{Mg}{m3} = 1.00\ g/mL$$

$$\gamma mercurry = SGmercurry.\gamma water = 13.56\ (9.81) = 133\ kN/m3.$$

$$\rho mercurry = SGmercurry.\rho water = 13.56\ (1.00) = 13.56\frac{Mg}{m3}$$
$$= 1.00\ g/mL$$

Note: N = kg. m/s^2 , kN=Mg.m $/s^2$, M (mega)=10^6

H.W/ The specific weight of water at ordinary pressure and temperature is 62.4 lb/ft^3 . The specific gravity of mercury is 13.56. Compute the density of water and the specific weight and density of mercury in BG unit.

3. Temperature (*T*): It is the measure of hotness and coldness of a system. There are different scales of temperature are expressed in Celsius /centigrade (°C), Fahrenheit (°F), absolute value in Kelvin (K), and Rankine (°R)

$K = {}^{o}C$+273.16 (in the SI system, Kelvin is expressed)

${}^{o}R = {}^{o}F$+459.69

4. Viscosity (*μ*): When two solid bodies in contact, move relative to each other, a friction force develops at the contact surface in the direction opposite to motion. The situation is similar when a fluid moves relative to a solid or when two fluids move relative to each other. The property that represents the internal resistance of a fluid to motion (i.e. *fluidity*) is called as ***viscosity***. The shear stress (τ) is expressed as,

$$\boldsymbol{\tau = \mu\frac{du}{dy}}$$

Where, $\frac{du}{dy}$ is the shear strain rate

μ is the dynamic (or absolute) viscosity of the fluid

The dynamic viscosity (μ) has the dimension ($ML^{-1}T^{-1}$) and the unit of kg/m.s (or, N.s/m^2 or Pa.s) . A common unit of dynamic viscosity is *poise* which is equivalent to 0.1 Pa.s. Many a times, the ratio of dynamic viscosity to density appears frequently and this ratio is given by the name kinematic viscosity ($\nu = \frac{\mu}{\rho}$)

The dimension of viscosity is L^2T^{-1} and the unit is *stoke* (1 stoke = 0.0001 m^2/s).

Typical values of kinematic viscosity of air and water at atmospheric temperature are 1.46 x 10^{-5} m^2/s and 1.14 x 10^{-6} m^2/s, respectively.

A substance in liquid / gas phase is referred as 'fluid'. Distinction between a solid & a fluid is made on the basis of substance's ability to resist an applied shear (tangential) stress that tends to change its shape. A solid can resist an applied shear by deforming its shape whereas a fluid deforms continuously under the influence of shear stress, no matter how small is its shape. **In solids, stress is proportional to strain, but in fluids, stress is proportional to 'strain rate.'**

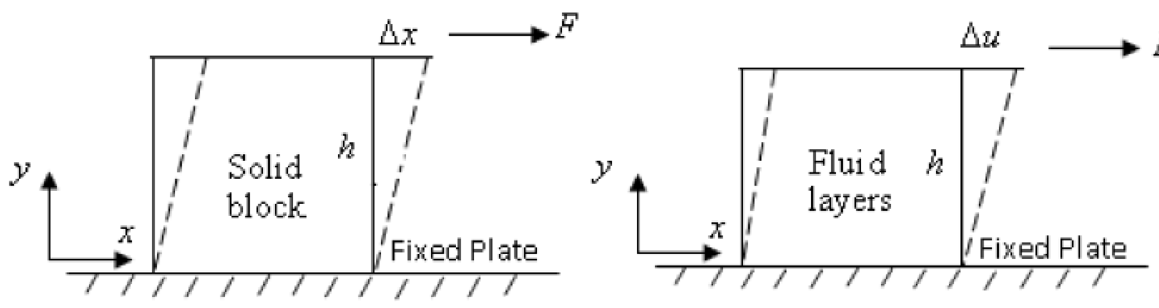


Fig.2.2: Illustration of solid and fluid deformation.

Referring to Fig. 2.2, the shear modulus of solid S and coefficient of viscosity μ for fluid can defined in the following manner;

$$S = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{(F/A)}{(\Delta x/h)}; \quad \mu = \frac{\text{Shear stress}}{\text{Shear strain rate}} = \frac{(F/A)}{(\Delta u/h)}$$

The shear force (F) is acting on the certain cross-sectional area (A) , h is the height of the solid block / height between two adjacent layer of the fluid element, Δx is the elongation of the solid block and Δu is the velocity gradient between two adjacent layers of the fluid.

In general, the viscosity of a fluid mainly depends on temperature. **For liquids, the viscosity decreases with temperature and for gases, it increases with temperature. Sutherland's** correlation is used to determine viscosity of **gases** as a function of temperature.

For air, the reference value of viscosity $\mu° = 1.789x10-5\ kg/m.s$ at $T°$

Sutherland's constant for gas, S = 110 k = 199 °R

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{\frac{3}{2}} \left(\frac{T_0 + S}{T + S}\right) \qquad \text{Sutherland's Law}$$

In the cases of **liquid**, the viscosity is approximated as below;

For water at $T° = 273\ k,\ \mu° = 0.001792\ kg/m.s,\ a = -1.94,\ b = -4.8,\ c = 6.74$

5. **Thermal Conductivity (k)**: It relates the rate of heat flow per unit area ($q^{\cdot}$) to the temperature gradient $\left(\frac{dT}{dx}\right)$ and is governed by **Fourier Law** of heat conduction i.e. $q^{\cdot} = -k\frac{dT}{dx}$

In SI system the unit and dimension of thermal conductivity can be written as, W/m.K and $MLT^{-3}\Theta^{-1}$, respectively. Thermal conductivity varies with temperature for liquids as well as gases in the same manner as that of viscosity. The reference value of thermal conductivity (***k_o***)for water and air at reference temperature is taken as, 0.6 W/m.K and 0.025 W/m.K, respectively.

6. Coefficient of compressibility/Bulk modulus (*Ev*): It is the property of the fluid that represents the variation of density with pressure at constant temperature. The compressibility (change in volume due to change in pressure) of a **liquid** is inversely proportional to its ***volume modulus of elasticity,*** also known as the ***bulk modulus.*** This modulus is defined as

$$E_v = -v\frac{dp}{dv} = -\left(\frac{v}{dv}\right)dp$$

where *v* is specific volume and *p is* pressure. As *v*/*dv* is a dimensionless ratio, the units of *Ev* and *p* are identical. The bulk modulus is analogous to the modulus of elasticity for solids; however, for fluids it is defined on a volume basis rather than in terms of the familiar one-dimensional stress–strain relation for solid bodies.

The bulk modulus of water does not vary a great deal for a moderate range in pressure. By rearranging the definition of *E*v, as an approximation we may use for the case of a fixed mass of liquid at constant temperature

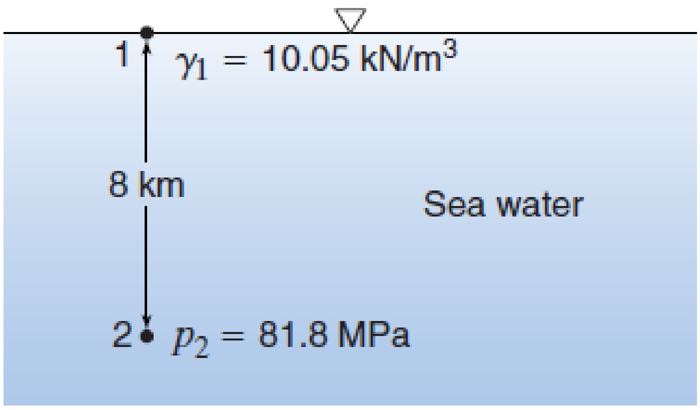
$$\frac{\Delta v}{v} \approx -\frac{\Delta p}{E_v}$$

$$\frac{v_2 - v_1}{v_1} \approx -\frac{p_2 - p_1}{E_v}$$

where Ev is the mean value of the modulus for the pressure range and the subscripts 1 and 2 refer to the before and after conditions.

Example 2.2: At a depth of 8 km in the ocean the pressure is 81.8 MPa. Assume that the specific weight of seawater at the surface is 10.05 kN/m^3 and that the average volume modulus is 2.34 109 N/m^2 for that pressure range. (*a*) What will be the change in specific volume between that at the surface and at that depth? (*b*) What will be the specific volume at that depth? (*c*) What will be the specific weight at that depth?

Solution



$$\frac{\Delta v}{v} \approx -\frac{\Delta p}{E_v}$$

$$\frac{v_2 - v_1}{v_1} \approx -\frac{p_2 - p_1}{E_v}$$

(a)

$$\nu 1 = {}^{1}/_{\rho 1} = {}^{g}/_{\gamma 1} = {}^{9.81}/_{10050} = 0.000976\ {}^{m3}/_{kg}$$

$$\Delta\nu = -0.000976\ (81.8\text{x}10^{6}-0)/(2.34\text{x}10^{9}) = -34.1\text{x}10^{-6}\ \text{m}^3/\text{kg}$$

(b)

$$\nu 2 = \nu 1 + \Delta\nu = 0.000942\ \ m3/kg$$

(c)

$$\gamma 2 = g/\nu 2 = 9.81/0.000942 = 10410\ \ N/m3$$

7. Coefficient of volume expansion (β): It is the property of that fluid that represents the variation of density with temperature at constant pressure. Mathematically, it is represented as,

$$\beta = \frac{1}{\cancel{v}}\left(\frac{\partial \cancel{v}}{\partial T}\right)_p = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

In terms of finite change, it is approximated as,

$$\beta = \frac{\left(\Delta \cancel{v} / \cancel{v}\right)}{\Delta T} = -\frac{\left(\Delta \rho / \rho\right)}{\Delta T}$$

8. Specific heats: It is the amount of energy required for a unit mass of a fluid for unit rise in temperature. Since the pressure, temperature and density of a gas are interrelated, the amount of heat required to raise the temperature from T_1 to T_2 depends on whether the gas is allowed to expand during the process so that the energy supplied is used in doing the work instead of raising the temperature. For a given gas, two specific heats are defined corresponding to the two extreme conditions of constant volume and constant pressure.

(a) Specific heat at constant volume ($\boldsymbol{Cv}$)

(b) Specific heat at constant pressure ($\boldsymbol{Cp}$)

The following relation holds good for the specific heat at constant volume and constant pressure. For **air** ; C_p = 1.005 KJ/kg.K , Cv = 0.718 KJ/kg. K

$$c_p - c_v = R; \quad c_p = \frac{\gamma R}{\gamma - 1}; \quad c_v = \frac{R}{\gamma - 1}$$

Where, R is gas constant

9. Speed of sound (*c*) : An important consequence of compressibility of the fluid is that the disturbances introduced at some point in the fluid propagate at finite velocity. The velocity at which these disturbances propagate is known as "acoustic velocity/speed of sound". Mathematically, it is represented as below;

$$c = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{E_v}{\rho}}$$

$$E_v = p \quad \Rightarrow c = \sqrt{\frac{p}{\rho}}$$

In an isothermal process

$$c = \sqrt{RT}$$ for an ideal gas

$$E_v = \gamma p \quad \Rightarrow c = \sqrt{\frac{\gamma p}{\rho}}$$

In isentropic process

$$c = \sqrt{\gamma RT}$$ for an ideal gas

10. Vapour pressure (p_v): It is defined as the pressure exerted by its vapour in phase equilibrium with its liquid at a given temperature. For a pure substance, it is same as the saturation pressure. In a fluid motion, if the pressure at some location is lower than the vapour pressure, bubbles start forming. This phenomenon is called as **cavitation** because they form cavities in the liquid.

11. Surface Tension (σ) : When a liquid and gas or two immiscible liquids are in contact, an unbalanced force is developed at the interface stretched over the entire fluid mass. The intensity of molecular attraction per unit length along any line in the surface is called as surface tension. For example, in a spherical liquid droplet of radius (r), the pressure difference (Δp) between the inside and outside surface of the droplet is given by,

$$\Delta p = \frac{2\sigma}{r}$$

State Relations for Gases

All gases at high temperatures and low pressures are in good agreements with 'perfect gas law' given by,

$$p = \rho RT = \rho\left(\frac{\bar{R}}{M}\right)T$$

where, $\boldsymbol{R}$ is the characteristic gas constant,

$\bar{\boldsymbol{R}}$ is the universal gas constant,

T is absolute temperature in K or °R,

M is the molecular weight

Since

$$\gamma = \rho g$$

So

$$\gamma = \frac{gp}{RT}$$

AlKarkh University of Science
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Fluid Mechanics

Stage-2

LECTURE THREE PRESSURE AND FLUID STATICS

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2022

Pressure

Pressure is defined as a normal force exerted by a fluid (gas or liquid) per unit area.

$$P = \frac{F}{A} \qquad 1\ pa = 1\ N/m^2$$

$1\ bar = 10^5\ pa = 100\ kpa$

$1\ atm = 101325\ pa = 1.01325\ bar = 14.696\ psi\ (lbf / in^2)$

Three types of pressures are:

- **Absolute pressure:** The actual pressure at a given position and it is measured relative to absolute vacuum (i.e., absolute zero pressure).
- **Gauge pressure:** the difference between the absolute pressure and the local atmospheric pressure.

$$P_{gage} = P_{abs} - P_{atm}$$

- **Vacuum pressure:** Pressures below atmospheric pressure and are measured by vacuum gages that indicate the difference between the atmospheric pressure and the absolute pressure.

$$P_{vac} = P_{atm} - P_{abs}$$

Ex.3.1: A vacuum gage connected to a chamber reads 5.8 psi at a location where the atmospheric pressure is 14.5 psi. Determine the absolute pressure in the chamber.

Solution: $P_{abs} = P_{atm} - P_{vac} = 14.5 - 5.8 = 8.7\ psi$

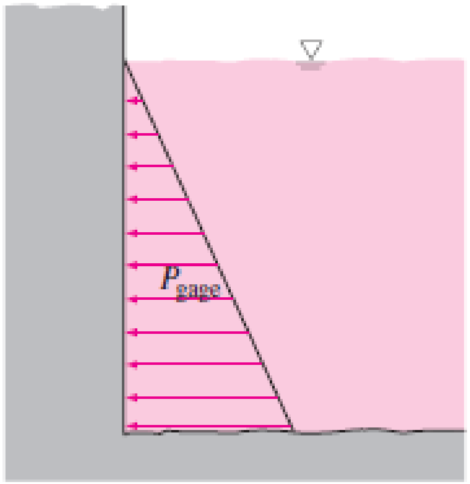


Figure 3.2: the pressure of a fluid at rest increases with depth (as a result of added weight)

Pressure in a continuously distributed uniform static fluid varies only with vertical distance and is independent of the shape of the container. The pressure is the same at all points on a given horizontal plane in the fluid. The pressure increases with depth in the fluid.

If we take point 1 to be at the free surface of a liquid open to the atmosphere (Fig. 3.5), where the pressure is the atmospheric pressure P_{atm}, then the pressure at a depth h from the free surface becomes, (Eq. 3. 3)

$$P = P_{atm} + \rho g h \qquad \text{or} \qquad P_{gage} = \rho g h$$

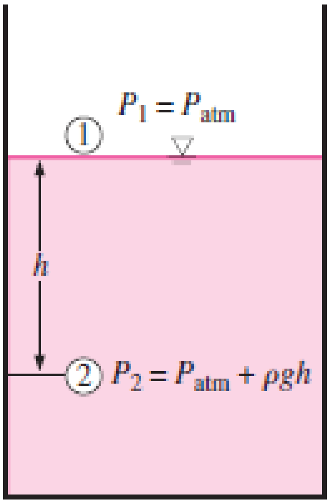


Figure 3.5: pressure in a liquid at rest increases linearly with distance from the free surface

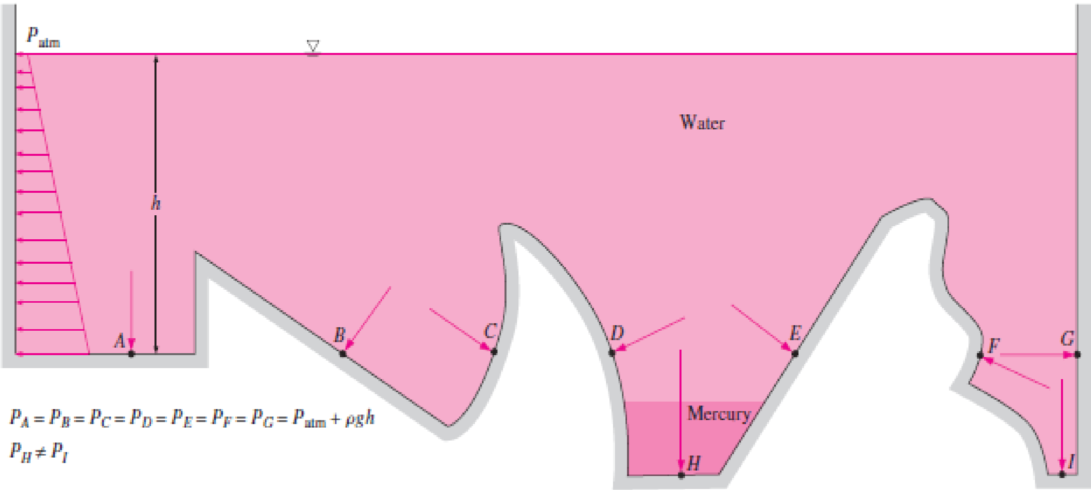


Figure 3.6: the pressure is the same all points on a horizontal plane in a given fluid regardless of geometry provided that the points are fluid interconnected by the same fluid

The area ratio A2 /A1 is called the ideal mechanical advantage of the hydraulic lift. Using a hydraulic car jack with a piston area ratio of A2 /A1 10, for example, a person can lift a 1000-kg car by applying a force of just 100 kgf (908 N).

$$P_1 = P_2 \quad \rightarrow \quad \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \rightarrow \quad \frac{F_2}{F_1} = \frac{A_2}{A_1}$$

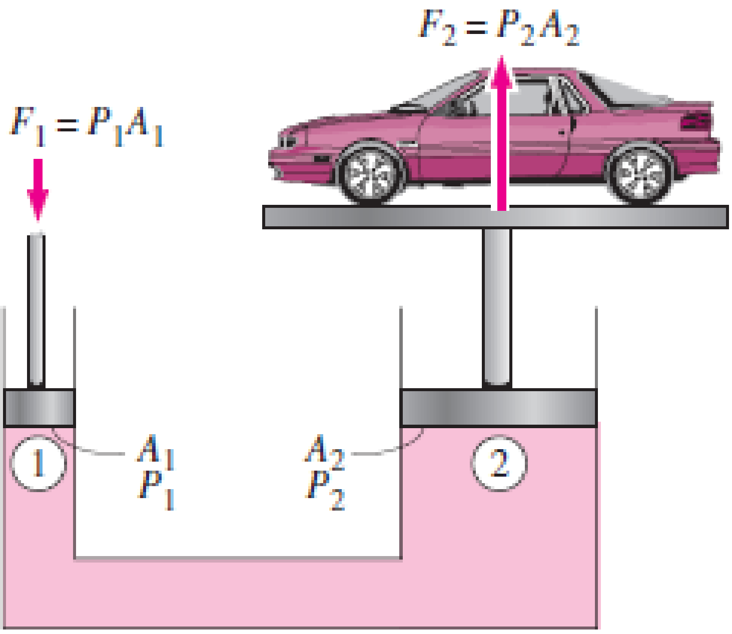


Figure 3.7: lifting of a large weight by a small force by the application of Pascal’s law

<u>Pressure Measurement</u>

- Bourdon Tube
- Barometer
- Manometer

a- <u>Bourdon Tube</u>

Named after the French engineer and inventor Eugene Bourdon (1808–1884), which consists of a hollow metal tube bent like a hook whose end is closed and connected to a dial indicator needle (Fig.3.8).

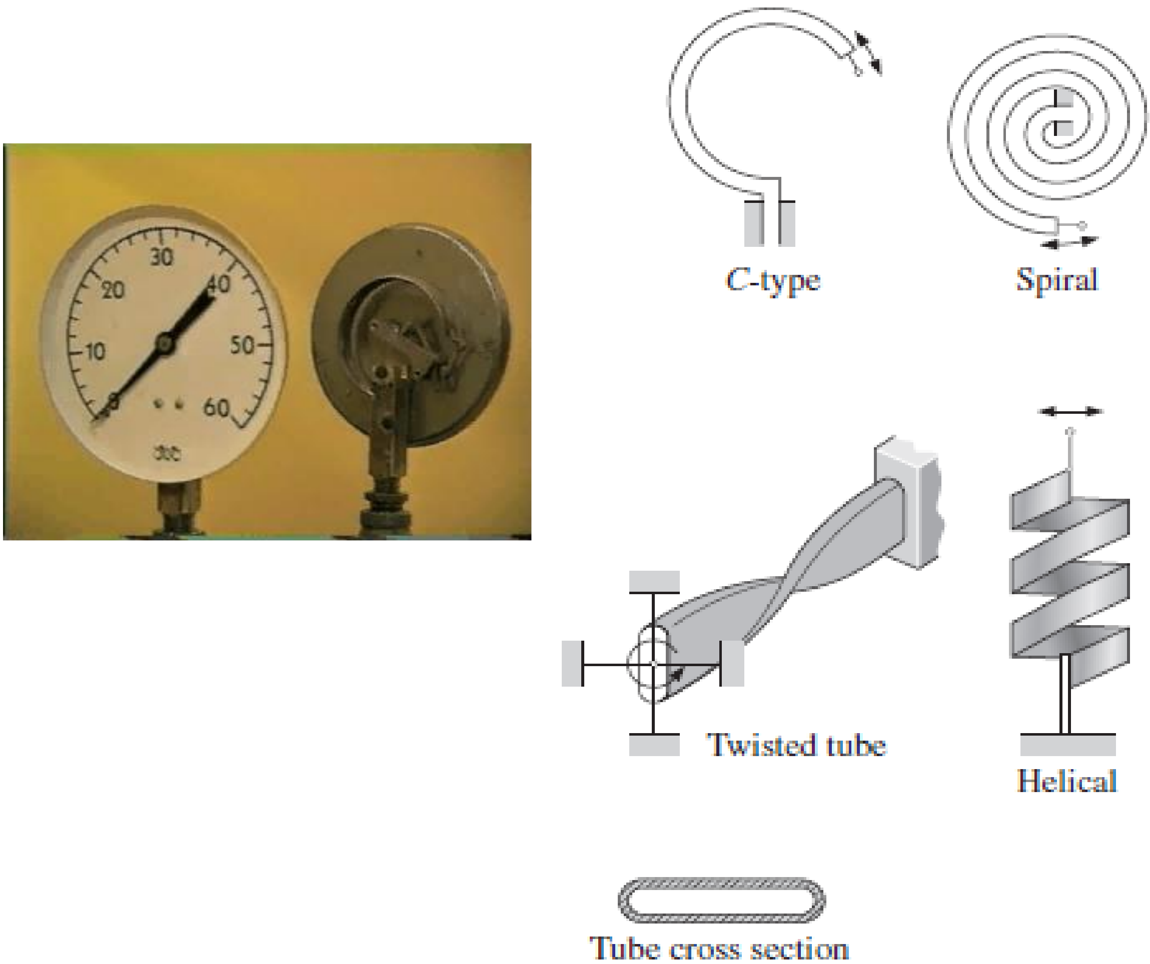


Figure 3.8: Various types of Bourdon tubes used to measure pressure

b-Barometer

Atmospheric pressure is measured by a device called a **barometer;** thus, the atmospheric pressure is often referred to as the *barometric pressure*.

$$P_{atm} = \rho g h$$

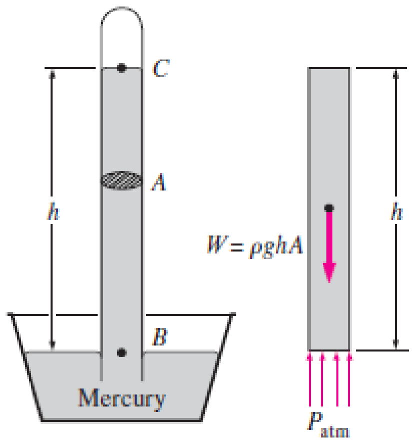


Figure 3.9; The basic barometer

where ρ is the density of mercury, *g* is the local gravitational acceleration, and *h* is the height of the mercury column above the free surface. Note that the length and the cross-sectional area of the tube have no effect on the height of the fluid column of a barometer (Fig. 3.10).

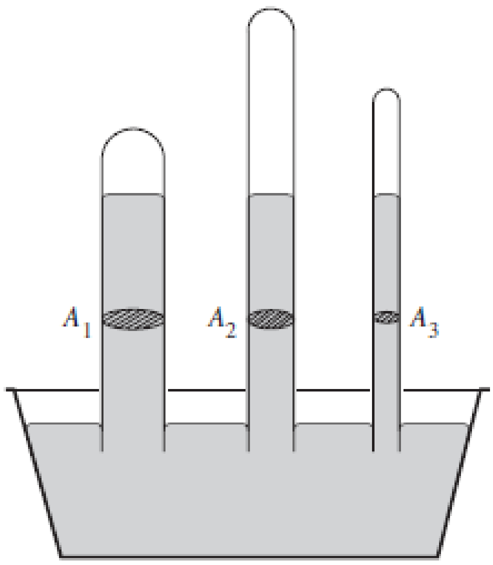


Figure 3.10: The length or the cross-sectional area on the height of the fluid column of a barometer, provided that the tube diameter is large enough to avoid surface tension (capillary) effect.

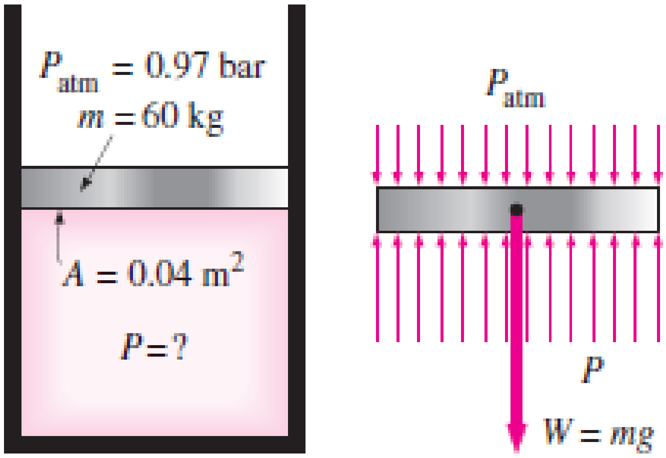
A frequently used pressure unit is the *standard atmosphere*, which is defined as the pressure produced by a column of mercury 760 mm in height at 0°C (ρ_{Hg}= 13,595 kg/m^3) under standard gravitational acceleration (g = 9.807 m/s^2). If water instead of mercury were used to measure the standard atmospheric pressure, a water column of about 10.3 m would be needed. Pressure is sometimes expressed (especially by weather forecasters) in terms of the height of the mercury column. The standard atmospheric pressure, for example, is 760 mmHg (29.92 inHg) at 0°C. The unit mmHg is also called the **torr** in honor of Torricelli. Therefore, 1 atm = 760 torr and 1 torr =133.3 Pa.

Ex.3.1 Determine the atmospheric pressure at a location where the barometric reading is 740 mm Hg and the gravitational acceleration is g = 9.81 m/s^2. Assume the temperature of mercury to be 10°C, at which its density is 13,570 kg/m^3.

Solution:

$$\begin{aligned} P_{\text{atm}} &= \rho g h \\ &= (13{,}570\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.74\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) \\ &= 98.5\ \text{kPa} \end{aligned}$$

Ex.3.2 The piston of a vertical piston–cylinder device containing a gas has a mass of 60 kg and a cross-sectional area of 0.04 m^2, as shown in Figure below. The local atmospheric pressure is 0.97 bar, and the gravitational acceleration is 9.81 m/s^2. (*a*) Determine the pressure inside the cylinder. (*b*) If some heat is transferred to the gas and its volume is doubled, do you expect the pressure inside the cylinder to change?



Solution: A gas is contained in a vertical cylinder with a heavy piston. The pressure inside the cylinder and the effect of volume change on pressure are to be determined.

Assumptions Friction between the piston and the cylinder is negligible.

Analysis (a) The gas pressure in the piston–cylinder device depends on the atmospheric pressure and the weight of the piston. Drawing the free-body diagram of the piston as shown in Figure above and balancing the vertical forces yield

$$PA = P_{atm}A + W$$

Solving for P and substituting,

$$P = P_{atm} + \frac{mg}{A}$$

$$= 0.97 \text{ bar} + \frac{(60 \text{ kg})(9.81 \text{ m/s}^2)}{0.04 \text{ m}^2}\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ bar}}{10^5 \text{ N/m}^2}\right)$$

$$= \mathbf{1.12 \text{ bars}}$$

(b) The volume change will have no effect on the free-body diagram drawn in part (a), and therefore the pressure inside the cylinder will remain the same.

c- Manometer

A device based on this principle is called a **manometer,** and it is commonly used to measure small and moderate pressure differences. A manometer mainly consists of a *glass* or *plastic* U-tube containing one or more fluids such as *mercury, water, alcohol, or oil*. To keep the size of the manometer to a manageable level, heavy fluids such as mercury are used if large pressure differences are anticipated. Consider the manometer shown in Fig. 3.11 that is used to measure the pressure in the tank. Since the gravitational effects of gases are negligible, the pressure anywhere in the tank and at position 1 has the same value. Furthermore, since pressure in a fluid does not vary in the horizontal direction within a fluid, the pressure at point 2 is the same as the pressure at point 1,

$$P_2 = P_1.$$

The differential fluid column of height h is in static equilibrium, and it is open to the atmosphere. Then the pressure at point 2 is determined directly from Eq. 3.3 to be,

$$P_2 = P_{atm} + \rho g h$$

where ρ is the density of the fluid in the tube. Note that the cross-sectional area of the tube has no effect on the differential height h, and thus the pressure exerted by the fluid. However, the diameter of the tube should be large enough (more than a few millimeters) to ensure that the surface tension effect and thus the capillary rise is negligible.

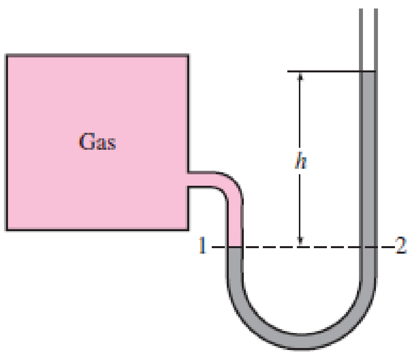
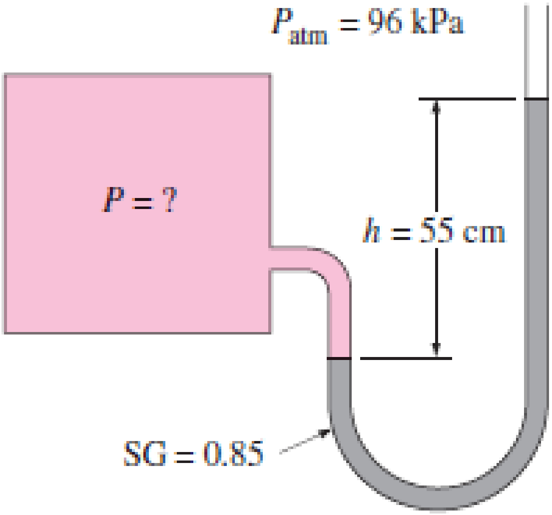


Figure 3.11: The basic manometer

<u>Ex.3.3</u> A manometer is used to measure the pressure in a tank. The fluid used has a specific gravity of 0.85, and the manometer column height is 55 cm, as shown in figure below. If the local atmospheric pressure is 96 kPa, determine the absolute pressure within the tank.



Solution: The reading of a manometer attached to a tank and the atmospheric pressure are given. The absolute pressure in the tank is to be determined.

Assumptions The fluid in the tank is a gas whose density is much lower than the density of manometer fluid.

ρ= SG (ρH_2O) = (0.85)(1000 kg/m^3) = 850 kg/m^3

$P = P_{atm} + \rho gh$

$$p = 96\ kpa + (850\ \ kg/m3)\ (9.81\ \ m/s2)(0.55\ m)(\frac{1\ N}{1\ kg.m/s2})(\frac{1\ kpa}{1000\ \ N/m2})$$

P = 100.6 kpa

Many engineering problems and some manometers involve multiple immiscible fluids of different densities stacked on top of each other. Such systems can be analyzed easily by remembering that (1) the pressure change across a fluid column of height h is $\Delta P = \rho gh$, (2) pressure increases downward in a given fluid and decreases upward (i.e., $P_{bottom} > P_{top}$), and (3) two points at the same elevation in a continuous fluid at rest are at the same pressure. The last principle, which is a result of *Pascal's law,* allows us to "jump" from one fluid column to the next in manometers without worrying about pressure change as long as we don't jump over a different fluid, and the fluid is at rest. Then the pressure at any point can be determined by starting with a point of known pressure and adding or subtracting r*gh* terms as we advance toward the point of interest. For example, the pressure at the bottom of the tank in Fig. 3.12 can be determined by starting at the free surface where the pressure is P_{atm}, moving downward until we reach point 1 at the bottom, and setting the result equal to $P1$. It gives

$$Patm + \rho 1gh1 + \rho 2gh2 + \rho 3gh3 = p1 \qquad \text{(Eq.3.7)}$$

In the special case of all fluids having the same density, this relation (Eq.3.7) reduces Eq. 3.3, as expected.

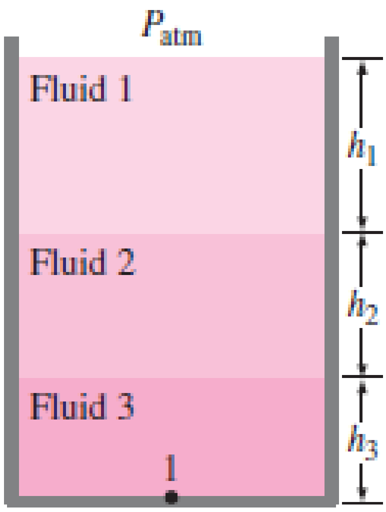


Figure 3.12: In stacked up fluid layer, the pressure change across a fluid layer of density ρ and height h is ρgh.

Manometers are particularly well-suited to measure pressure drops across a horizontal flow section between two specified points due to the presence of a device such as a valve or heat exchanger or any resistance to flow. This is done by connecting the two legs of the manometer to these two points, as shown in Fig.

3.13. The working fluid can be either a gas or a liquid whose density is ρ_1. The density of the manometer fluid is ρ_2, and the differential fluid height is h.

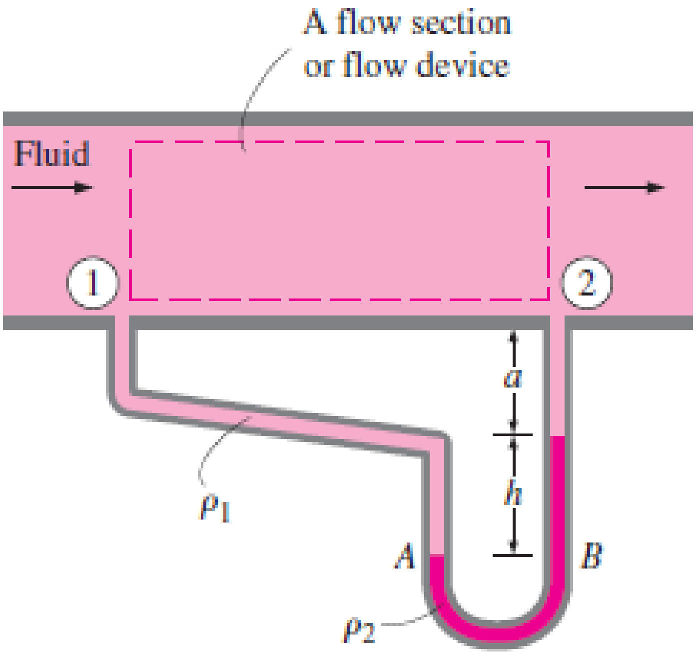


Figure 3.13: Measuring the pressure drop a cross a flow section or a flow device by a differential manometer

A relation for the pressure difference P_1- P_2 can be obtained by starting at point 1 with P_1, moving along the tube by adding or subtracting the ρgh terms until we reach point 2, and setting the result equal to P_2:

$$p1 + \rho 1g(a + h) - \rho 2gh - \rho 1ga = p2$$

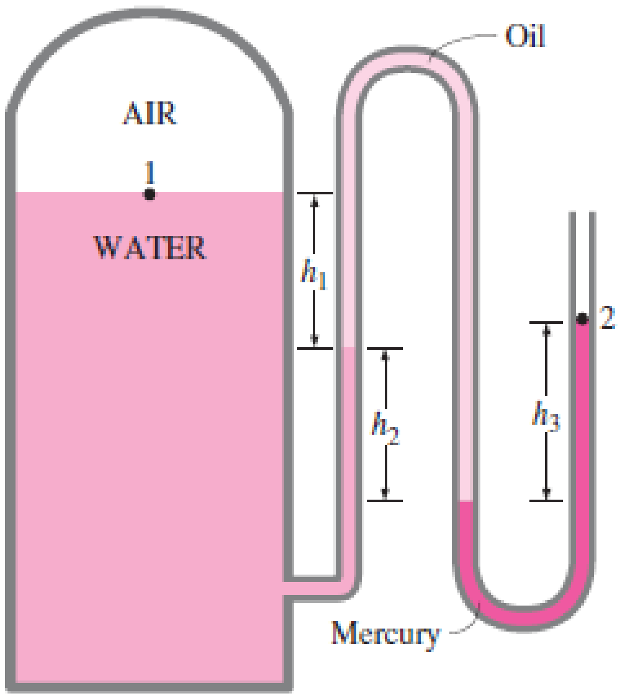
Note that we jumped from point ***A*** horizontally to point ***B*** and ignored the part underneath since the pressure at both points is the same. Simplifying,

$p1 - p2 = (\rho 2 \; - \; \rho 1)gh$ (Eq.3.8)

Note that the distance ***a*** has no effect on the result, but must be included in the analysis. Also, when the fluid flowing in the pipe is a gas, then $\rho_1 << \rho_2$ and the relation in Eq.3.8 simplified to $p_1 - p_2 \approx \rho_2 gh$

H.W:

The water in a tank is pressurized by air, and the pressure is measured by a multi fluid manometer as shown in Figure below. The tank is located on a mountain at an altitude of 1400 m where the atmospheric pressure is 85.6 kPa. Determine the air pressure in the tank if $h_1 = 0.1$ m, $h_2 = 0.2$ m, and $h_3 = 0.35$ m. Take the densities of water, oil, and mercury to be 1000 kg/m^3, 850 kg/m^3, and 13,600 kg/m^3, respectively.



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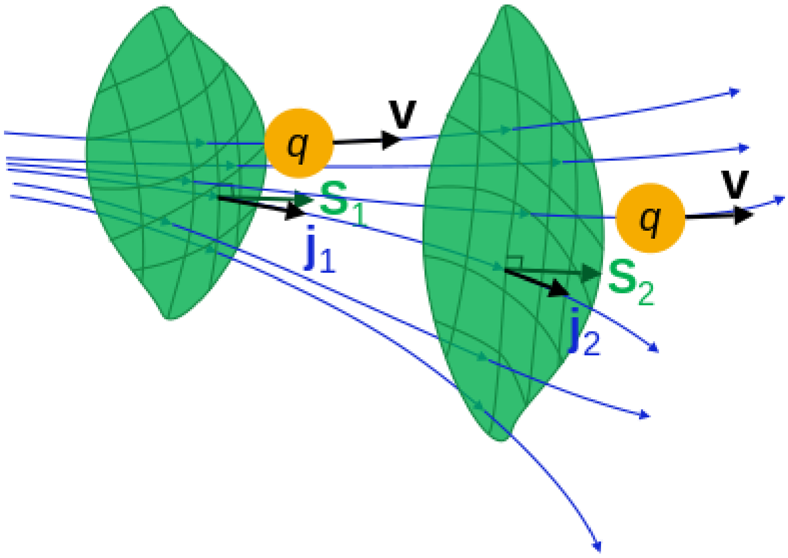


Fluid Mechanics

Stage-2

LECTURE FOUR
CLASSIFICATION OF FLUID FLOW, CONTROL VOLUME, CONTINUITY EQUATION

Dr. Ali K. Resen

2022

Classification of fluid flow

1. Uniform and non-uniform flow
2- steady and unsteady flow
3- Viscous and inviscid flow
4- Incompressible and compressible flows
5- Laminar and turbulent flows

1. Uniform and non-uniform flow

- **Non-Uniform flow:** If we look at a fluid flowing under normal circumstances -a river for example -the conditions (e.g., velocity, pressure) at one point will vary from those at another point.

  If at a given instant, the velocity is not the same at every point the flow is **non-uniform**
- **Uniform flow:** If the flow velocity is the same magnitude and direction at every point in the flow it is said to be uniform. That is, the flow conditions

  DO NOT change with position.

Reference:

YUNUS, A.C. & JOHN M. C. (2018) *Fluid Mechanics: Fundamental Applications , United State of American,* McGraw-Hill

2. **Steady and unsteady flow**

- **Steady**: A steady flow is one in which the conditions (velocity, pressure and cross-section) may differ from point to point but **DO NOT change with time.**
- **Unsteady**: If at any point in the fluid, the conditions change with time, the flow is described as unsteady.

Combining the above (1 & 2) we can classify any flow in to one of four types:

- **Steady uniform flow**. Conditions do not change with **position in the stream or with time**. An example is the flow of water in a pipe of constant diameter at constant velocity.
- **Steady non-uniform flow.** Conditions change from point to point in the stream but **do not change with time**. An example is flow in a tapering pipe with constant velocity at the inlet -velocity will change as you move along the length of the pipe toward the exit.
- **Unsteady uniform flow.** At a given instant in time the conditions at every point are the same, but will **change with time**. An example is a pipe of constant diameter connected to a pump pumping at a constant rate which is then switched off.
- **Unsteady non-uniform flow.** Every condition of the flow may change from point to point and with time at every point. An example is surface waves in an open channel.

3- Viscous and inviscid flow

▪ **viscous flow:** the effects of viscosity are important and cannot be ignored.

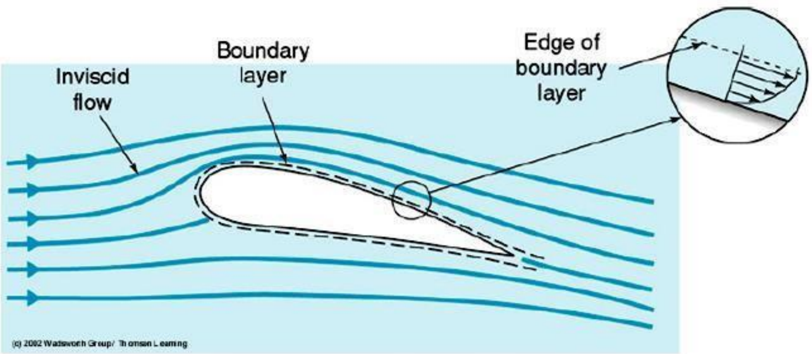
• **Inviscid flow**: an **inviscid flow** is one in which viscous effects do not significantly influence the flow and are thus neglected.

If the shear stresses in a flow are small and act over such small areas that they do not significantly affect the flow field the flow can be assumed as inviscid flow.

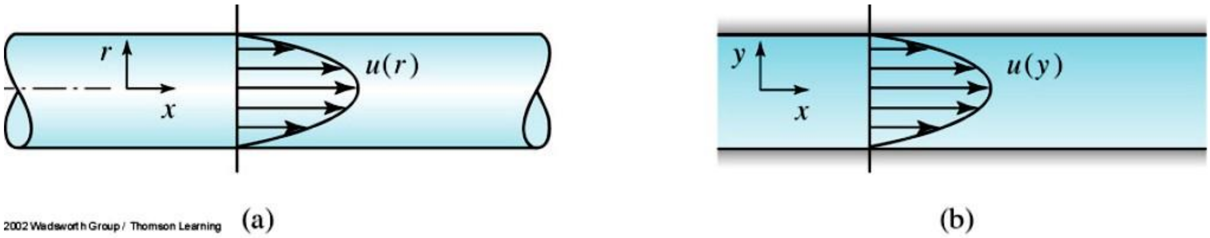
✓ It has been found that the primary class of flows, which can be modeled as inviscid flows, is **external flows,** that is, flows of an unbounded fluid which exist exterior to a body. Any viscous effects that may exist are confined to a thin layer, called a **boundary layer,** which is attached to the boundary**.**

✓ The velocity in a boundary layer is always zero at a fixed wall, a result of viscosity.

✓ For many flow situations, boundary layers are so thin that they can simply be ignored when studying the gross features of a flow around a streamlined body.



External flow around an airfoil.



Viscous internal flow: (a) in a pipe; (b) between two parallel plates.

4- Incompressible and compressible flows

- All fluids are **compressible** even water their density will change as pressure changes
- Under steady conditions, and provided that the **changes in pressure are small**, it is usually possible to simplify analysis of the flow by assuming it is **incompressible** and has constant density
- As you will appreciate, liquids are quite difficult to compress so under most steady conditions they are treated as incompressible. In some unsteady conditions very high pressure differences can occur and it is necessary to take these into account even for liquids
- Gases, on the contrary, are very easily compressed, it is essential in cases of high speed flow to treat these as compressible taking changes in pressure into account
- Low speed gas flows, such as the atmospheric flow referred to above, are also considered to be incompressible flows.

The Mach number is defined as

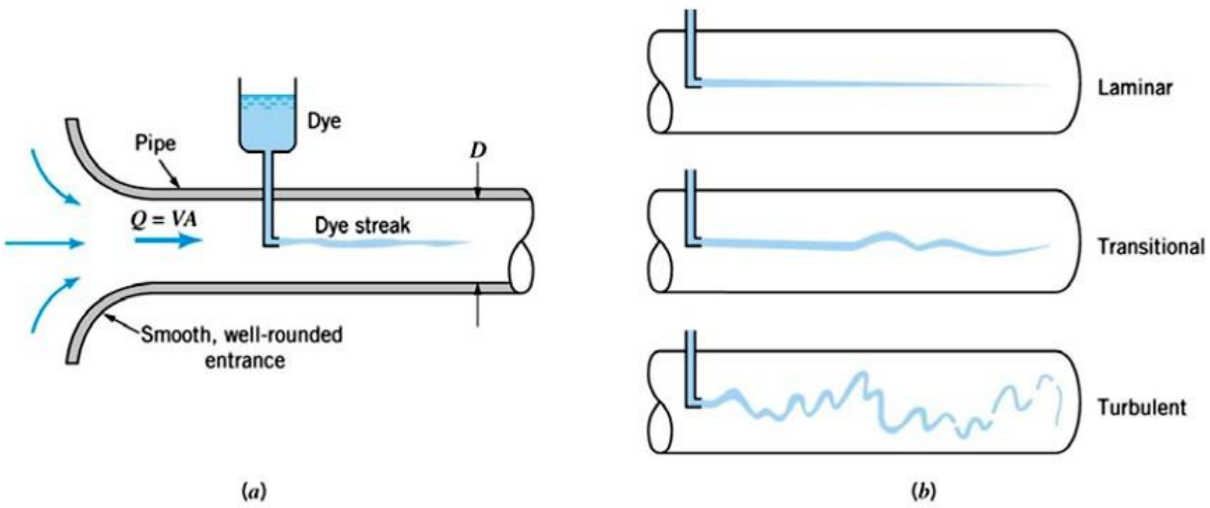
$$\mathbf{M} = \frac{\boldsymbol{V}}{\boldsymbol{C}}$$

where **V** is the gas speed and **c** is the speed of sound.

The Mach number is useful in deciding whether a particular gas flow can be studied as an incompressible flow.

- If **M < 0.3** , density variations are at most 3 % and the flow is assumed to be incompressible; for standard air this corresponds to a velocity below about 100 m/s.
- If **M > 0.3** , the density variations influence the flow and compressibility effects should be accounted for.

5- Laminar and turbulent flows



In the experiment shown above, a dye is injected into the middle of pipe flow of water. The dye streaks will vary, as shown in (b), depending on the flow rate in the pipe.

The top situation is called **laminar flow**, and the lower is **turbulent flow**, occurring when the flow is sufficiently **slow** and **fast**, respectively.

- In laminar flow the motion of the fluid particles is very orderly with all particles moving in straight lines parallel to the pipe wall. There is essentially no mixing of neighboring fluid particles.

- In sharp contrast, mixing is very significant in turbulent flow, in which fluid particles move haphazardly in all directions.

- It is therefore impossible to trace motion of individual particles in turbulent flow.

Whether the flow is laminar or not depends on the **Reynolds number**,

$$\mathrm{Re} \equiv \frac{\rho \bar{V} d}{\mu}$$

$\rho =$ density, $\mu =$ viscosity, $\bar{V} =$ section-mean velocity, $d =$ diameter of pipe

and it has been demonstrated experimentally that

$$\mathrm{Re} \begin{cases} < 2{,}000 & \text{laminar flow} \\ \text{between 2,000 and 4,000} & \text{transitional flow} \\ > 4{,}000 & \text{turbulent flow} \end{cases}$$

Basic Physical Laws of Fluid Mechanics

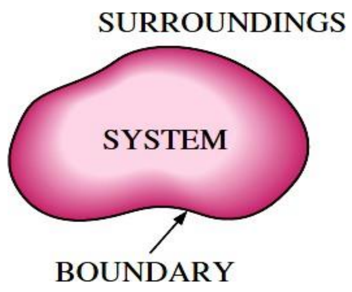
Systems and Control Volumes

A system is defined as a quantity of matter or a region in space chosen for study.

- The mass or region outside the system is called the **surroundings.**
- The real or imaginary surface that separates the system from its surroundings is called the **boundary.**

The boundary of a system can be **fixed** or **movable**.

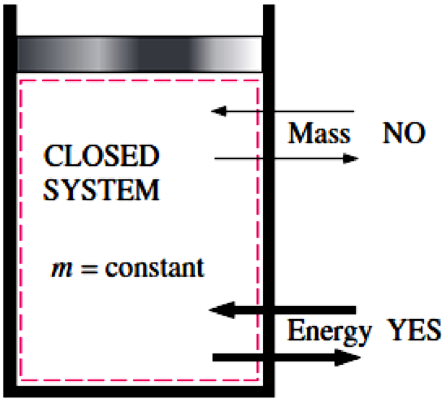
Note that the boundary is the contact surface shared by both the system and the surroundings. Mathematically speaking, the boundary has zero thickness, and thus it can neither contain any mass nor occupy any volume in space.



Systems and Control Volumes

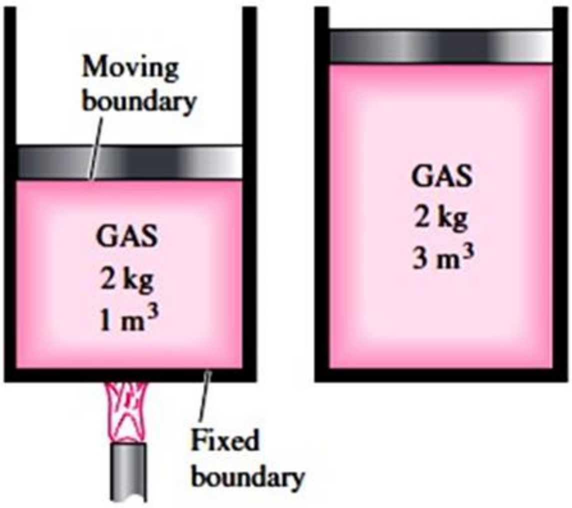
•Systems may be considered to be *closed or open,* depending on whether a fixed mass or a fixed volume in space is chosen for study.

•A **closed system** (also known as a **control mass)** consists of a fixed amount of mass, and no mass can cross its boundary. That is, no mass can enter or leave a closed system.



•But energy, in the form of heat or work, can cross the boundary; and the volume of a closed system does not have to be fixed.

- If, as a special case, even energy is not allowed to cross the boundary, that system is called an isolated system.



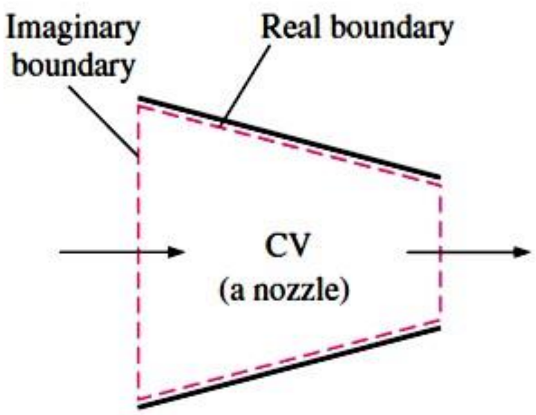
- An open system, or a control volume, as it is often called, is a properly selected region in space.
- A ***control volume*** usually encloses a device that involves ***mass flow*** such as a compressor, turbine, or nozzle. Flow through these devices is best studied by selecting the region within the device as the control volume. ***Both mass and energy can cross the boundary of a control volume.***
- A large number of engineering problems involve mass flow in and out of a system

and, therefore, are modeled as control volumes.

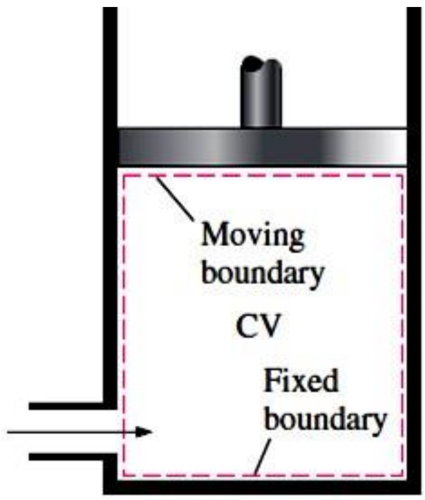
- A water heater, a car radiator, a turbine, and a compressor all involve mass flow and should be analyzed as control volumes (open systems) instead of as control masses (closed systems).
- In general, ***any arbitrary region in space can be selected as a control volume.*** There are no concrete rules for the selection of control volumes, but the proper choice certainly makes the analysis much easier.

Systems and Control Volumes (CV)

The boundaries of a control volume are called a ***control surface,*** and they can be ***real*** or ***imaginary***. In the case of a nozzle, the inner surface of the nozzle forms the real part of the boundary, and the entrance and exit areas form the imaginary part, since there are no physical surfaces there.



(*a*) A control volume with real and imaginary boundaries



(*b*) A control volume with fixed and moving boundaries

- The **laws of mechanics** state what happens when there is an interaction between the system and its surroundings.
- **First**, the system is a fixed quantity of mass, denoted by *m*. Thus, the ***mass of the system is conserved and does not change***. This is a law of mechanics and has a very simple mathematical form, called ***conservation of mass***:

$$m_{\text{syst}} = \text{const}$$

$$\frac{dm}{dt} = 0$$

- **Second**, if the surroundings exert a net force ***F*** on the system, **Newton's second** law states that the mass in the system will begin to accelerate

$$\mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{V}}{dt} = \frac{d}{dt}(m\mathbf{V})$$

- Newton's second law is called the linear momentum relation.
- Note that it is a vector law that implies the three scalar equations

$$F_x = ma_x,\ F_y = ma_y,\ \text{and } F_z = ma_z.$$

- **Third**, if the surroundings exert a net moment **M** about the center of mass of the system, there will be a rotation effect

$$\mathbf{M} = \frac{d\mathbf{H}}{dt}$$

where $\mathbf{H} = \sum(\mathbf{r} \times \mathbf{V})\ \delta m$ is the angular momentum of the system about its center of mass. This is called the **angular momentum relation**.

- **Fourth**, if heat δQ is added to the system or work δW is done by the system, the system energy *dE must change* according to ***the energy relation, or first law of thermodynamics:***

$$\delta Q - \delta W = dE$$

$$\dot{Q} - \dot{W} = \frac{dE}{dt}$$

Finally, the second law of thermodynamics relates entropy change ***dS to heat added dQ and absolute temperature T:***

$$dS \geq \frac{\delta Q}{T}$$

This is valid for a system and can be written in control volume form, but there are almost no practical applications in fluid mechanics

Elementary Equations of Fluid Motion

We shall derive the three basic control-volume relations in fluid mechanics:

1. The principle of conservation of ***mass***, from which the ***continuity equation*** is developed; 2.The principle of conservation of ***energy***, from which the energy equation is derived; 3.The principle of conservation of ***linear momentum***, from which equations evaluating dynamic forces exerted by flowing fluids may be established.

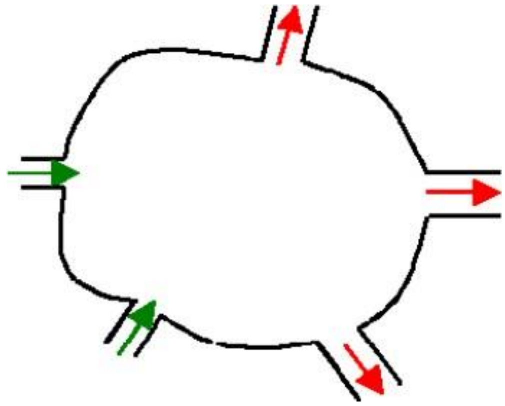
Control volume

- A control volume is a finite region, chosen carefully by the analyst for a particular problem, with open boundaries through which ***mass***, ***momentum***, and ***energy*** are allowed to cross.
- The analyst makes a *budget*, or *balance*, between the incoming and outgoing fluid

and the resultant changes within the control volume. Therefore one can calculate the gross properties (net force, total power output, total heat transfer, etc.) with this method.

- With this method, however, we do not care about the details inside the control volume.

- Let us consider a control volume that can be a tank, reservoir or a compartment inside a system, and consists of some definite one-dimensional inlets and outlets, like the one shown.



- Let us denote for each of the inlets and outlets: -

  $V =$ *velocity of fluid in a stream*

  $A =$ *sectional area of a stream*

  $p =$ *pressure of the fluid in a stream*

Then, the volume flow rate, or discharge (volume of flow crossing a section per unit time) is given by

$$Q = VA$$

- Similarly, the mass flow rate (mass of flow crossing a section per unit time) is given by

$$\dot{m} = \rho VA = \rho Q$$

Then, the momentum flux, defined as the momentum of flow crossing a section per unit time, is given by $\dot{m}V$

Continuity equation

- By steadiness, the total mass of fluid contained in the control volume must be invariant with time.

Therefore there must be an exact balance between the total rate of flow into the control volume and that out of the control volume:

Total Mass Outflow = Total Mass Inflow

which translates into the following mathematical relation

$$\sum_{i=1}^{M}(\rho_i V_i A_i)_{\text{in}} = \sum_{i=1}^{N}(\rho_i V_i A_i)_{\text{out}}$$

Where ***M*** is the number of inlets, and ***N*** is the number of outlets.

- If the fluid is incompressible, e.g. water, with ρ being effectively constant, then .

$$\sum_{i=1}^{M}(V_i A_i)_{\text{in}} = \sum_{i=1}^{N}(V_i A_i)_{\text{out}} \quad \text{or} \quad \sum_{i=1}^{M}(Q_i)_{\text{in}} = \sum_{i=1}^{N}(Q_i)_{\text{out}}$$

$$Q = V_1 A_1 = V_2 A_2 = \text{constant}$$

Example 1. Water Flow through a Garden Hose Nozzle

A garden hose attached with a nozzle is used to fill a 10-gal bucket. The inner diameter of the hose is 2 cm, and it reduces to 0.8 cm at the nozzle exit. If it takes 50 s to fill the bucket with water, determine:

(a)the volume and mass flow rates of wate through the hose, and

(b) the average velocity of water at the nozzle exit

SOLUTION A garden hose is used to fill a water bucket. The volume and mass flow rates of water and the exit velocity are to be determined.

Assumptions **1** Water is an incompressible substance. **2** Flow through the hose is steady. **3** There is no waste of water by splashing.

Properties We take the density of water to be 1000 kg/m³ = 1 kg/L.

Analysis (*a*) Noting that 10 gal of water are discharged in 50 s, the volume and mass flow rates of water are

$$\dot{V} = \frac{V}{\Delta t} = \frac{10 \text{ gal}}{50 \text{ s}}\left(\frac{3.7854 \text{ L}}{1 \text{ gal}}\right) = \mathbf{0.757\ L/s}$$

$$\dot{m} = \rho\dot{V} = (1 \text{ kg/L})(0.757 \text{ L/s}) = \mathbf{0.757\ kg/s}$$

(*b*) The cross-sectional area of the nozzle exit is

$$A_e = \pi r_e^2 = \pi(0.4 \text{ cm})^2 = 0.5027 \text{ cm}^2 = 0.5027 \times 10^{-4} \text{ m}^2$$

The volume flow rate through the hose and the nozzle is constant. Then the average velocity of water at the nozzle exit becomes

$$V_e = \frac{\dot{V}}{A_e} = \frac{0.757 \text{ L/s}}{0.5027 \times 10^{-4} \text{ m}^2}\left(\frac{1 \text{ m}^3}{1000 \text{ L}}\right) = \mathbf{15.1\ m/s}$$